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Discrete-Ordinates Solution of Radiative Transfer Equation in Nonaxisymmetric Cylindrical Enclosures

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The S_4 discrete-ordinates approximation is used to solve the radiative transfer equation in nonasymmetric (i.e., three-dimensional) cylindrical enclosures containing absorbing-emitting and scattering media, with and without the temperature profile known a priori. Because neither detailed experimental data nor predictions from a zone or Monte-Carlo model for three-dimensional cylindrical enclosures are available, cylindrical equivalents of three-dimensional rectangular enclosures, for which zone model predictions of radiative transfer are available, are used in model evaluation. Limited evaluation of the model shows that the discrete-ordinates method provides acceptable predictions of radiative transfer in nonaxisymmetric cylindrical enclosures.

Nomenclature

A_F = total area of the furnace, m²
A, B, C = areas in the radial, axial, and azimuthal directions, m²
f = differencing factor
k_a, k_s, k_t = absorption, scattering and extinction coefficients, m⁻¹
I = radiation intensity, kW/(m²·sr)
$P(\Omega, \Omega')$ = scattering phase function
q = hemispherical flux, kW/m²
R = furnace radius, m
r, z = distance along radial and axial directions, m
S_{nr} = nonradiative source term, kW/m³
V_F = total volume of the furnace enclosure, m³
v_p = volume of the computational cell, m³
w_m = angular quadrature weight, sr
ε = emissivity
Θ = space variable in the azimuthal direction, rad
ξ, μ, η = direction cosines
ϕ = azimuthal angle, rad
ω = albedo of scatter
Ω, Ω' = outward and inward directions of radiation

Subscripts
b = black body
i, j, k = area indices in the axial, radial, and azimuthal directions
m, m' = discrete directions
o = initial value
r, z = radial and axial
w = wall

Superscripts
$+, -$ = positive and negative directions of propagation

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Introduction

THE major part of the modeling effort for pulverized coal combustors to-date has been directed toward modeling axisymmetric cylindrical systems. Two major reasons for this are that laboratory and pilot-scale combustors for experimentation under controlled test conditions have been mostly cylindrical in configuration, and that the assumption of axial symmetry greatly reduces the complexity of the computations resulting in significant savings in computer time. Unfortunately, even these simplified combustors are often not truly axisymmetric. In many practical systems of cylindrical configuration the assumption of axisymmetry introduces but small error. However, in cases where tertiary inlets are used or where asymmetric intrusions are present, or uneven heating of the combustor walls is involved, the assumption of symmetry may cause rather large errors.

The present work describes and evaluates a prediction method that is an extension of techniques used for predicting radiative transfer in axisymmetric cylindrical,[1] and two- and three-dimensional rectangular[2] enclosures; and accounts for the lack of symmetry in the azimuthal direction in a cylindrical system. This is perhaps the first effort in modeling radiative transfer in nonaxisymmetric cylindrical enclosures using the discrete-ordinates method (see also Jamaluddin and Smith[3]). Other publications dealing with nonaxisymmetric cylindrical enclosures include Crosbie and Farrell[4] and Yucel and Williams.[5]

Formulation

The equation of transfer for the radiation intensity in a discrete direction in a three-dimensional cylindrical system is written as

$$\frac{\mu_m}{r}\frac{\partial(rI_m)}{\partial r} + \frac{\eta_m}{r}\frac{\partial I_m}{\partial \Theta} + \xi_m\frac{\partial I_m}{\partial Z} - \frac{1\partial}{r}\frac{(n_m I_m)}{\partial \phi}$$
$$= -(k_a + k_s)I_m + k_a I_b$$
$$+ \frac{k_s}{4\pi}\int_{4\pi} P\left(\underset{\sim}{\Omega}, \underset{\sim}{\Omega}'\right) I_{m'}\, \mathrm{d}\underset{\sim}{\Omega}' \tag{1}$$

The left side of Eq. (1) represents the gradient of intensity in the direction of propagation, and the right side represents, respectively, the attenuation of intensity due to absorption and out-scattering, and the contribution to the directional intensity due to emission by the medium, and in-scattering.

$P(\Omega, \Omega')$, the phase function, determines the distribution of the scattered intensity. $\xi_{m'}$, $\mu_{m'}$, and η_m are the direction cosines for the discrete direction Ω.

In S_4 discrete-ordinates approximation, the transfer equation is solved in 24 directions spanning the total solid angle (previous work[2] has shown that S_4 approximation is adequate for three-dimensional systems). The angular integral is evaluated using numerical gradrature. Carlson and Lathrop[6] give details of the discrete-ordinates theory and the solution technique for neutron transport.

Assuming the surrounding surfaces to be diffuse and opaque, Eq. (1) is solved with the following boundary conditions

$$\begin{aligned}
&\text{at } r = R: I_m = \varepsilon_w I_{bw} + (1 - \varepsilon_w)\frac{q_r^+}{\pi}; \qquad \mu_m < 0\\
&\text{at } r = 0: I_m = I_{m'}, \mu_m = \mu_{m'}\\
&\text{at } z = 0: I_m = \varepsilon_w I_{bw} + (1 - \varepsilon_w)\frac{q_z^-}{\pi}; \qquad \xi_m > 0\\
&\text{at } z = L: I_m = \varepsilon_w I_{bw} + (1 - \varepsilon_w)\frac{q_z^+}{\pi}; \qquad \xi_m < 0
\end{aligned} \tag{2}$$

with the hemispherical fluxes obtained as:

$$q_d = \int_{2\pi}\left(\underset{\sim}{n}\cdot\underset{\sim}{\Omega}'\right) I \, d\underset{\sim}{\Omega}' \tag{3}$$

The angular derivative in Eq. (1) is evaluated using the direct-differencing technique of Carlson and Lathrop.[6]

Equation (1) with given boundary conditions [Eq. (2)] is solvable provided the temperature of the medium is known. In situations where the temperature of the medium is an unknown, an additional equation, the radiation energy balance equation, needs to be solved

$$S_{nr} = 4\pi k_a I_b - k_a \int_{4\pi} I \, d\underset{\sim}{\Omega}' \tag{4}$$

The first and the second terms on the right side of Eq. (4) represent the radiation emitted by, and incident on, the volume element, respectively. S_{nr} is the rate at which energy is generated within the volume element by means other than radiation. Rewriting Eq. (1) in terms of S_{nr}

$$\begin{aligned}
&\frac{\mu_m}{r}\frac{\partial(rI_m)}{\partial r} + \frac{\eta_m}{r}\frac{\partial I_m}{\partial \Theta} + \xi_m \frac{\partial I_m}{\partial z} - \frac{1}{r}\frac{\partial(\eta_m I_m)}{\partial \phi}\\
&= -(k_a + k_s)I_m + \frac{1}{4\pi}\left[S_{nr} + k_a \int_{4\pi} I_{m'} \, d\underset{\sim}{\Omega}'\right.\\
&\left.+ k_s \int_{4\pi} P\left(\underset{\sim}{\Omega}, \underset{\sim}{\Omega}'\right) I_{m'} \, d\underset{\sim}{\Omega}'\right]
\end{aligned} \tag{5}$$

Multiplying both sides of Eq. (1) by $r\,dr\,dz\,d\Theta$, and integrating over the volume element gives

$$\begin{aligned}
&\xi_m(A_{i+1}I_{m,i+1} - A_i I_{m,i}) + \mu_m(B_{j+1}I_{m,j+1} - B_j I_{m,j})\\
&+ \eta_m(C_{k+1}I_{m,k+1} - C_k I_{m,k}) - (B_{j+1} - B_j)\\
&\cdot\frac{\alpha_{m+1/2}I_{m+1/2} - \alpha_{m-1/2}I_{m-1/2}}{w_m}\\
&= -v_p(k_a + k_s)I_m + v_p k_a I_b + k_s v_p I_s
\end{aligned} \tag{6}$$

The intensities $I_{m,i+1}$, $I_{m,j+1}$ and $I_{m,k+1}$ can be expressed in terms of the discrete intensities $I_{m,i}$, $I_{m,j}$ and $I_{m,k}$ using weighted diamond-differencing

$$I_{i+1} + fI_i = I_{j+1} + fI_j = I_{k+1} + fI_k = (1 + f)I_m \tag{7}$$

I_m, the intensity at the center of the volume element can therefore be evaluated as

$$I_m = \frac{\xi_m A I_i + \mu_m B I_j + \eta_m C I_k + \dfrac{B_j - B_{j+1}}{w_m}\alpha I_\phi + k_a v_p I_b + k_s v_p I_s}{\xi_m A + \mu_m B + \eta_m C + \dfrac{B_j - B_{j+1}}{w_m}\alpha + (k_a + k_s)v_p} \tag{8}$$

where:

$$\begin{aligned}
A &= A_i + fA_{i+1}\\
B &= B_j + fB_{j+1}\\
C &= C_k + fC_{k+1}\\
\alpha &= \alpha_{m-1/2} + f\alpha_{m+1/2}\\
I_s &= \sum_{m'} P\left(\underset{\sim}{\Omega}, \underset{\sim}{\Omega}\right) I_{m'} w_{m'}/\pi
\end{aligned}$$

Solution Procedure

The solution procedure involves solving the transfer equation in each of the ordinate directions, forming a set of 24 coupled partial differential equations. The calculation is started at the $z = L$ and $r = R$ wall, at the azimuthal location where the boundary condition is known (in a partially transparent medium, the calculation may start at any Θ-location whereby an initial estimate of the azimuthal component of intensity is obtained by solving Eq. (1) in the special direction where $\eta_m = 0.0$, assigning zero weight to the solution so obtained). The axis serves as a transparent boundary. The directions of traverse are so chosen that the values of the direction cosines gradually increase, a change in the sign of the direction cosine signifying a reversal in the corresponding direction of integration.

The solution of the discrete-ordinates equations must be obtained iteratively, as the calculated intensities enter the boundary conditions, and the in-scattering term. Once the intensity at the center of a volume element is known [from Eq. (8)], that at the downstream surface can be obtained by extrapolation using Eq. (7). However, the central differencing scheme (i.e., $f = 1.0$) used in Eq. (8) often results in negative intensities (particularly in the presence of steep gradients, or where spatial resolution is not high enough). If negative intensities are encountered during the calculation, the global value of f [see Eq. (7)] is gradually decreased from its initial value of 1.0 until the negativity is removed. For $f = 0.0$, the intensity is always non-negative, and special directions are not required to initiate the recursive solution.

Evaluation

The accuracy of the discrete ordinates solutions depends on the choice of the quadrature scheme. Although this choice is, in principle, arbitrary, completely symmetric quadratures are preferred in order to preserve geometric invariance of the solutions. The quadrature scheme used in the present work is based on the "moment-matching" technique of Carlson and Lathrop,[6] and is chosen to satisfy the zero- and second-order full-range (i.e., 4π) moments, as well as first-order half-range (i.e., 2π) moments of intensity distribution. The values of the ordinates are available elsewhere.[2,7,8]

Before proceeding on to evaluate the method for cylindrical equivalents of three-dimensional rectangular enclosures, the method was evaluated by predicting the wall heat flux in an axisymmetric cylindrical furnace, treating it as nonasymmetric. Figure 1 shows a sketch of the furnace,[9] as well as the predictions of the zone and the discrete-ordinates models (both 2- and 3-D calculations). The two- and three-dimensional cylindrical calculations used 17×18 and $17 \times 18 \times 8$ grid-resolution, respectively. Identical predictions from two- and three-dimensional calculations show that, in the limit, the 3-D cylindrical model duplicates the predictions of the two-dimensional (i.e., axisymmetric) cylindrical calculations. The calculations for the two-dimensional enclosure required 6 s of computer time on a VAX 11/750 computer, while for the three-dimensional calculations are computational time was twenty-times more.

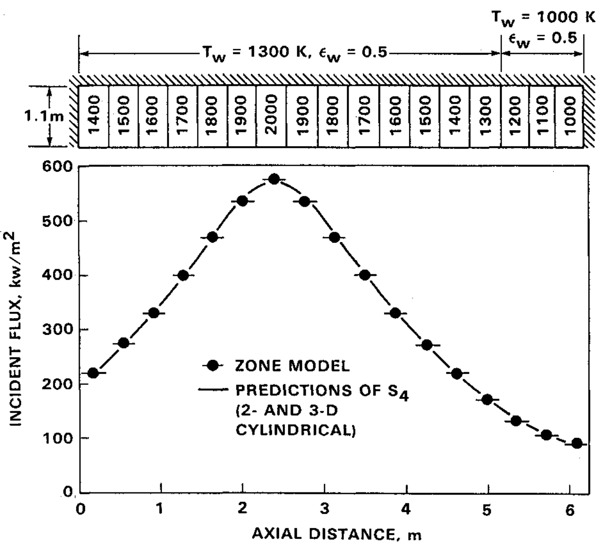

Fig. 1 Comparison of 3-D cylindrical model predictions with zone[9] and discrete-ordinates model predictions for an axisymmetric cylindrical furnace.

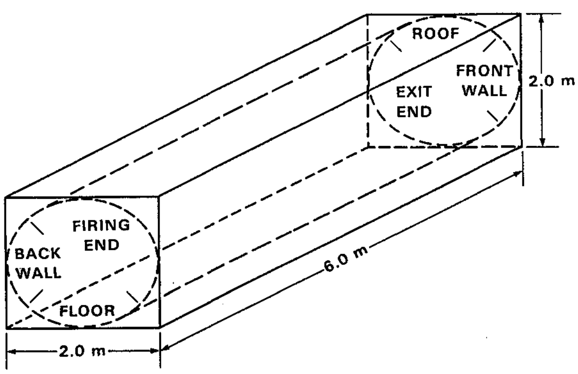

Fig. 2 Rectangular enclosure and its cylindrical equivalent.

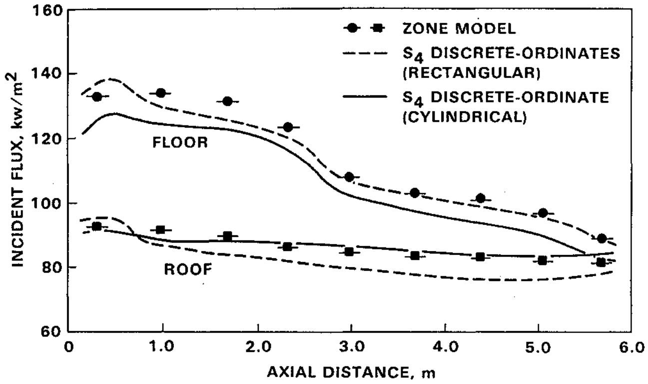

Fig. 3 Incident heat fluxes to the floor and the roof of the IFRF furnace (see Table 1) predicted by 3-D cylindrical model (for the equivalent cylindrical furnace), and the zone[10] and discrete ordinates[2] models (for the original rectangular furnace).

Table 1 Description of the 3-D furnace used in the IFRF M_3 trials[10]

Furnace dimensions	$6.0 \text{ m} \times 2.0 \text{ m} \times 2.0 \text{ m}$
Wall temperatures and emissivities	Floor, $T = 320.0$ K, $\varepsilon_w = 0.86$ Others, $T = 1090.0$ K, $\varepsilon_w = 0.70$
Properties of the medium	$k_a = 0.2 \text{ m}^{-1}$. T - measured

Table 2 Description of the 3-D furnace of Menguc and Viskanta[11]

Furnace dimensions	$4.0 \text{ m} \times 2.0 \text{ m} \times 2.0 \text{ m}$
Wall temperatures and emissivities	Firing-end, $T = 1200.0$ K, $\varepsilon_w = 0.85$ Exit-end, $T = 400.0$ K, $\varepsilon_w = 0.70$ Others, $T = 900.0$ K, $\varepsilon_w = 0.70$
Properties of medium	$k_t = 0.5 \text{ m}^{-1}$, $S_{nr} = 5.0 \text{ kW/m}^3$, $\omega = 0.70$

Enclosures with Known Temperature Profiles

The test-case (IFRF M3 trials, Flame 10) used in this evaluation has been used earlier to evaluate the discrete-ordinates method for three-dimensional rectangular enclosures.[2] The dimensions of the actual rectangular furnace (described in Table 1), and its cylindrical equivalent used in the present evaluation are shown in Fig. 2. The cylindrical equivalent has been chosen as one that has the same length and V_F/A_F ratio. The hypothetical furnace so obtained is somewhat smaller than the actual furnace, and the absolute values of the predicted wall heat fluxes would, therefore, be somewhat different. The temperatures of the medium are reported in Hyde and Truelove[10] and Jamaluddin and Smith.[2]

Comparison of the predictions of the three-dimensional cylindrical calculations with the predictions of the zone and discrete-ordinates models for the rectangular enclosure, depicted in Fig. 3, indicates that the nonaxisymmetric cylindrical calculations are quite accurate, given that the dimensions of the furnaces are a little different. The grid resolution used in the rectangular furnace calculations is $18 \times 6 \times 6$, while that used in the cylindrical furnace calculations is $18 \times 3 \times 8$.

Enclosures with Temperature Profiles Unknown

An absorbing-emitting-scattering medium with uniformly distributed nonradiative heat source is studied under this category. This test case has been studied earlier by Menguc and Viskanta.[11] The zone model predictions are taken from Truelove.[12] The discrete-ordinates predictions for the rectangular enclosure are from Jamaluddin and Smith.[2] Detailed information about the enclosure is provided in Table 2.

Figure 4 compares the predictions of the present calculations for an equivalent cylindrical enclosure with those of the

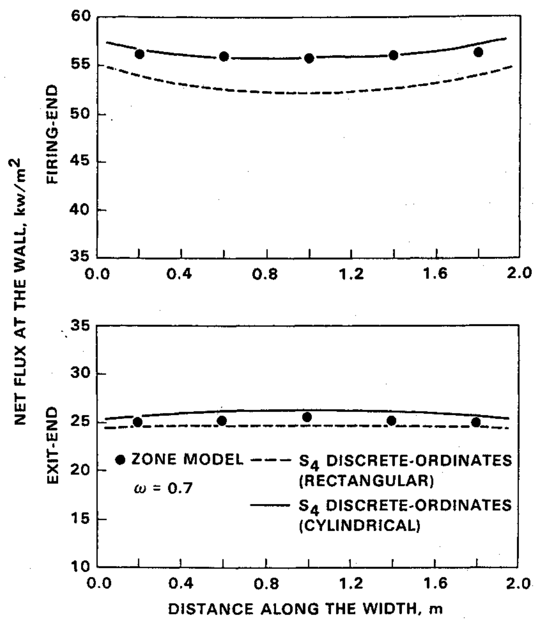

Fig. 4 Predictions of the 3-D version of the discrete-ordinates model compared with those of zone[12] and discrete-ordinates (rectangular)[2] models for the net wall radiative flux in an enclosure containing absorbing-emitting-scattering medium[11] (see Table 2).

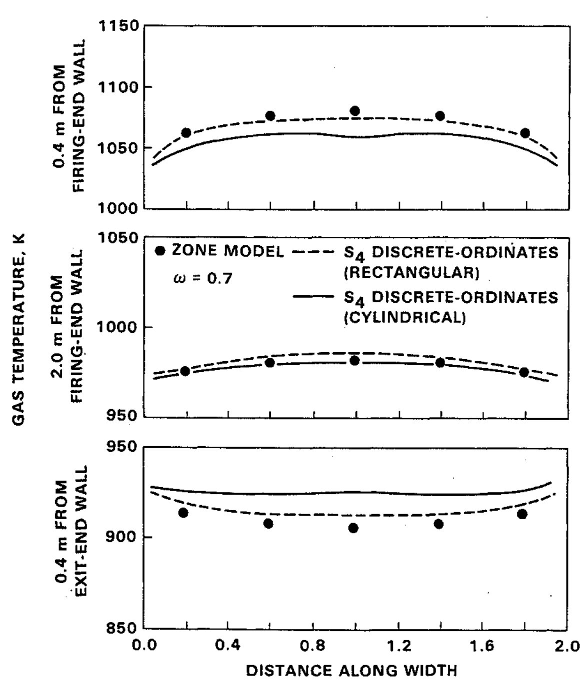

Fig. 5 Predictions of the 3-D version of the discrete-ordinates model compared with those of zone[12] and discrete-ordinates (rectangular)[2] models for the temperature of the medium in an enclosure containing absorbing-emitting-scattering medium[11] (see Table 2).

zone and discrete-ordinates models for the original rectangular enclosure. The discrete-ordinates calculations for the rectangular enclosure was done using 11 × 7 × 7 grid resolution, while the cylindrical furnace calculations used 11 × 5 × 8 grids. The three sets of predictions for the net heat fluxes at the firing- and exit-end walls are in very good agreement. The predicted temperature of the medium at three axial locations are also in excellent agreement (see Fig. 5).

Conclusions

This work is perhaps the first effort in modeling radiative transfer in three-dimensional cylindrical enclosures using the discrete-ordinates method. The limited evaluation presented here demonstrates the discrete-ordinates method to be a useful technique in predicting radiative transfer in nonaxisymmetric cylindrical systems.

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